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Ch 11: Quadric surfaces and Conics

Lab Purpose

One of the most important skills necessary for success in a Calculus C class is the ability to reason and imagine surfaces in 3 dimensions. In order to help build that skill, the purpose of this lab is to examine several different types of quadric surfaces and their properties.

Necessary Materials/Programs

1. Geogebra 3D Graphing calculator <https://www.geogebra.org/3d>
2. Desmos 2D Graphing calculator. <https://www.desmos.com/calculator>

Procedure

1. As part of our analysis of the 3D surfaces we will be looking at in this lab, we are going to analyze some simple 2 dimensional curves.
   1. Within desmos, let's start off simple. Graph what is this curve called? Describe this graph and how it behaves.
      1. Answer: Parabola. Lower y – limit of 0, upper y limit of infinity, x is all real numbers. Curve that has its point at 0,0.
   2. Now, graph . Select add slider for c. (if you want the previous graph to disappear, just click on the colored circle) Play around with various values of c and describe how the shape of the conic changes with c.This is called a hyperbola. Does the value of c affect the graph more near the origin or at the end? What is its end behavior?
      1. Answer: Affects more at the origin, jumps closer to 0,0 when you get closer to c = 0. The end behavior stays the same the further you get from the origin
   3. Finally, consider the ellipse. Define =1. Describe how changing b and c alters the ellipse.
      1. Answer: B changes the x direction of the ellipse and c changes the y direction of the ellipse. Further you get from 0 the larger the ellipse becomes. When |c| = |b| it becomes a circle
2. Now we can begin the analysis of rudimentary 3D quadric surfaces.
   1. First, within geogebra define the following: . Type in the entire equation and then click on “ Create Slider” (or just hit enter if that is not an option.) Define sliders for a,b, and c, and make sure to keep them all positive. You can change the default value of -5 to 0.1 by clicking on -5 and typing 0.1 then enter. How does altering the value of a affect the figure? What about b and c? Be specific.
      1. Answer: Starts as a sphere. a, b, and c respectively to x, y and z changes the axis of the ellipse / sphere. So if you changed a, the sphere will change into an ellipse along the x-axis. If you made all of them the same value the sphere’s radius will get bigger.
   2. Now, suppose you took a cross section of this figure along the plane z=0. What is the name of this figure. (hint, just set z=0 and analyze the equation). Now delete the plane z=0 and type x=0. What does this cross section look like? Now do the same for y=0. Are they all the same type of figure? By the way, these cross sections are called traces.
      1. Answer: the cross section is a circle/oval looking top down. If you set x equal to 0 and y you will get the same circle/oval, just looking from a different angle.
   3. From this, if you had to name this figure in part a, what would you call it. Be creative.
      1. Answer: Egg
   4. Now, for those of you who are curious, the official name for this figure is an ellipsoid.
3. Now keep the sliders for a, b, and c but delete or edit the previous equation of the ellipsoid. Type the equation Be sure to hit enter to get the new graph.
   1. What is this figure called?
      1. Answer: Elliptical cone
   2. In what ways would the figure be altered by varying the values of a,b, and c. Describe each case individually.
      1. Answer: a & b stretch / shrink the cone along their respective axes, and c changes the radius of the cone overall. Further, as c grows the radius shrinks.
   3. Now suppose you were to look at the cross section of the function along the planes x=1, y=1, and z=1. What would each of these figures be, based off of what we did in part A. If you need help, type in x=1 on a new line and see the cross section for yourself. You can repeat with y=1 and z=1. (do them one at a time, deleting the old one before adding a new one)
      1. Answer: The cross section of x = 1 & y = 1 create a parabola shape while the z = 1 equation makes a circle/oval as the cross section. For x = y & y = 1, it would be based on the x^2-y^2 = c equation from part a, and z = 1 would be the (x^2)/(b^2) + (y^2)/(c^2) = 1 (equation for a circle).
   4. The official name for this figure is an elliptical cone.
4. Now we move on to the surface described by . First delete or edit the surface equation created in part c. Change the slider for c so that c goes from -5 to 5.
   1. Like before, alter the values of a,b, and c and describe how these alterations affect the surface that appears. You might have to zoom out a little to see the whole graph. Do these effects make sense?
      1. Answer: Yeah, follows the same parameters. Changing a & b changes the x & y axes respectively. Changing c affects the radius and height of the Hyperboloid, and flips over the x axis when it approaches 0.
   2. Now, consider the figure that is formed if we took the cross section of this surface along the planes z=4, y=1, x=1. What figures are formed in each case.
      1. Answer: z = 4 makes a circle/oval, while x, y = 1 make another parabola shape.
   3. Now, based off of the plane curves that are formed by the cross section, what would you call this figure. Again be creative.
      1. Answer: Separated hourglass
   4. Finally, as a point of note, the official name for this surface is a hyperboloid of two sheets.
5. Now, consider the figure generated by . Keep the values of a and b positive but allow c to stay between -5 and 5.
   1. What do you think this figure is called?
      1. Answer: Cone
   2. Alter the values of a,b, and c. Describe how the figure changes in each case.
      1. Answer: a & b change x & y axis of the parabola while c changes the radius of the cone and flips over the flat plane at c = 0. The further from 0 the smaller the radius.
   3. What are the plane curves formed by the cross section along the planes x=0 and y=0. What is the plane curve formed by the intersection with the plane z=3?
      1. Answer: z = 3 creates a circle/oval while x, y = 0 make parabolas.
   4. This figure is called an elliptical paraboloid.
6. Now, consider the figure generated by . What does this figure look like?
   1. Predict how the figure and what it looks like will be altered by changing the values of a,b, and c. Next, within Geogebra, actually alter those values and see if your predictions were correct.
      1. Answer: a & b should change the x & y axis respectively, while c should change the radius of the figure.
   2. Now, take the cross sections formed by intersecting the figure with the planes x=0, y=0, and z=0. What figures are formed?
      1. Answer: x & y intersections create a perfect hourglass shape while z = 0 makes a circle/oval.
   3. Based off of the figures formed by this figure, what would you name this figure?
      1. Answer: Hourglass
   4. This figure is actually called a hyperboloid of one sheet.

G. Finally, consider the figure given by .

* 1. Play around with values of a,b, and c, making c positive again. Put your cursor on the graph and drag it around so that you can see the shape. Describe what each variable changing does to the graph and answer the following. Has the overall effect of changing each individual variable been the same throughout the whole lab? Why do you think this is?
     1. Answer: as a grows it spreads further away from the x axis, and as b grows it gets further from the y – axis. C changes the overall height of the figure and flips at 0. (As c grows the height grows).
  2. Consider the point 0,0,0 on the graph. This point is called a saddle point. Describe how the sheet actually behaves at this point.
     1. Answer: At exactly 0,0,0, the shape does not change at all. Should just be a dot or a small line. It changes completely at any point not 0,0,0, but at that specific point nothing changes.
  3. Finally, come up with a name for what you expect this figure to be called based off of the cross sections.
     1. Answer: x makes a parabola z>0 and y makes a parabola z<0. Any z = 0 cross section makes a hyperbola. Combination of a hyperbola and a parabola.
  4. For the curious, this figure is actually called a hyperbolic paraboloid.